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The Ex Ante Optimal Unemployment Insurance[*]

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Chul-In Lee
Seoul National University

Abstract: The optimal unemployment insurance (UI) literature initiated by the seminal work of Baily (1978) focuses on unemployed workers' moral hazard in job search conditional on "unemployment that has already occurred" – the *ex post* approach. In contrast, this study considers the optimal UI "before unemployment occurs" – the *ex ante* approach – by looking at moral hazard in work efforts of employed workers caused by generous benefits and the resulting firms' endogenous employment adjustment.

A notable advantage of taking this ex ante approach is that we can study the interaction between individuals and firms in response to UI benefit changes in a general equilibrium framework. New findings include: (i) labor supply elasticity is a central determinant of the optimal benefit level, more important than the risk aversion parameter; (ii) the optimal benefit level is also sensitive to the size of existing public expenditures; and (iii) the optimal level is lower than those in existing studies taking the ex post approach.

Key Words: social insurance, unemployment, moral hazard, ex ante optimality.

I. Introduction

In discussion of optimal unemployment insurance (UI), the literature addresses UI effect through the lens of the seminal theory of the optimal structure of UI by Baily (1978) in the following two aspects. First, it studies unemployed workers' responses to UI benefits. While there are some variations in empirical results, it is known that a 10% increase in UI benefits leads to a 1.2 weeks increase in compensated unemployment duration of unemployed job searchers (e.g., Meyer, 1990). Macroeconomic calibration studies of optimal UI adopt search or contract models, but they also focus on unemployed workers' search efforts (e.g., Davidson and Woodbury, 1997; Hopenhayn and Nicolini, 1997). Second, in the firm side the literature examines how firms respond to the imperfect experience rating system of UI (i.e., firms' UI contributions are less than proportional to firms' uses of UI through layoffs). While the imperfect experience rating system was developed for providing cost sharing insurance among firms, it is found to cause excessive layoffs, a firm-side moral hazard.[1] Overall, under a perfect experience rating system, the traditional views as to UI can be summarized by balancing unemployed workers' job search incentives with the original insurance effects of UI.[2]

However, if we consider optimal UI from the perspective of employed workers who may potentially experience unemployment depending upon their work efforts (the ex ante approach), we will be able to study the following two new aspects of optimal UI. First, when we approach

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[1] See Feldstein (1978) and Topel (1983) for the discussion about how imperfect experience rating affects firms' layoff behavior. The literature documents that a large portion of unemployment is due to firms' excessive layoffs in response to imperfect experience rating.

[2] There are many empirical studies showing that social insurance programs such as unemployment insurance (UI) reduce labor supply. For example, Moffitt (1985), Meyer (1990), and others have shown that a 10% increase in unemployment benefits raises average unemployment durations by 4-8% in the U.S. This finding has traditionally been interpreted as evidence of moral hazard caused by a substitution effect: UI distorts the relative price of leisure and consumption, reducing the marginal incentive to search for a job. For instance, Krueger and Meyer (2002, p2328) remark that behavioral responses to UI and other social insurance programs are large because they lead to short-run variation in wages with mostly a substitution effect. Similarly, Gruber (2007, p395) notes that UI has a significant moral hazard cost in terms of subsidizing unproductive leisure. Recently, Chetty (2008) presents another important element in the optimal design of UI, i.e., a liquidity effect of UI, both theoretically and empirically.

UI in the ex ante sense, we can easily see that generous UI leads to a reduced utility gap in utility between employed and unemployed workers, causing a low work incentive of workers at work. Second, in the general equilibrium context, we can also understand firms' endogenous behavior as to the low work incentive, i.e., profit-maximizing firms seek subsequent employment adjustment as a worker-disciplinary device to raise labor productivity (e.g., Shapiro and Stiglitz, 1984; Yellen, 1984). Especially in most modern economies with well-established social safety net, the problem addressed here may be particularly relevant. Of course some economies try to limit employment adjustment through regulations, but the presence of employment protection legislation can be seen as evidence of firms' potential demand for employment adjustment. Further, despite various restrictions, these economies allow implementation of labor policies, essentially equivalent to employment adjustment, such as allowing temporary employment contracts with no commitment of renewal, hiring a small portion of temporary/part-time workers as regular workers, applying strict hiring/screening procedures to minimize the likelihood of hiring less-motivated workers, etc.

In short, the literature focuses on moral hazard in job search in response to changes in UI policy parameters while disregarding moral hazard of workers at work and firms' behavioral responses to workers' moral hazard. In contrast to the existing literature that focuses on the UI system "after unemployment occurred" (ex post approach), our study considers the optimal UI system "before unemployment occurs" (the ex ante approach). While the existing approach is certainly appealing and shares lots of similarity with ours, our ex ante approach is particularly useful for policy purposes because an ideal design of UI should properly control the sources of unemployment in advance, which is usually thought to be a cost-effective way of handling unemployment.[3] Further, given that this study considers the environment where UI benefits are

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[3] A few papers take an approach that focuses on employment behavior as in our paper. For example, Acemoglu and

high enough to cause moral hazard, we may sidestep the liquidity effect issue that is studied in Chetty (2008) and can focus on the moral hazard of employed workers.

In our ex ante approach, we can examine the role of UI benefits in a general equilibrium model where (i) taking into account wages, unemployment benefits and taxes, workers choose work effort that the firm can observe only through costly monitoring; and (ii) firms use both wage and monitoring as effort-eliciting devices, and they fire those caught shirking. Therefore, the chosen level of a worker's effort affects the probabilities of employment and unemployment, and thus involuntary unemployment arises from moral hazard. In this contractual setting with imperfect information, we consider the optimal UI benefits and obtain some new insights.

Our analysis shows that firms' endogenous responses to worker moral hazard is crucial, accounting for a large share of the total change in unemployment in response to UI benefit changes. More specifically, it is notable that the labor supply elasticity is an essential parameter that affects the level of optimal UI benefits. With a high labor supply elasticity, moral hazard would be potentially large, so we need to implement a low benefit level and a low replacement ratio (benefit divided by earnings) in order to fight it. As in existing studies, risk aversion is still important as a determinant of the optimal benefit level. But we find through our analysis that the replacement ratio is not much responsive to the variation in the risk aversion parameter, suggesting the relative importance of the labor supply elasticity to the risk aversion parameter. Further, the existing market distortion is also an important determinant of the UI benefit level. If it is already high, e.g., due to high government provision of public goods, the shadow value of resources redistributed to unemployment consumption is high, so that the optimal benefit level should be set low. This result can only be obtained in a general equilibrium model as ours.

The basic story of our model is consistent with empirical regularities. Notwithstanding

Shimer's (1999) work on UI and labor market sorting across industries show that UI encourages workers to seek

the diverse nature of unemployment, a substantial part of unemployment in many developed countries, it is argued, stems from "generous" social welfare systems (Nickell and Layard, 1999; Meyer, 1990), suggesting the importance of controlling moral hazard not only ex post but also ex ante.[4] We acknowledge that the relevance of our model would be limited in accounting for the real world labor market when layoffs or firing workers is partially regulated by labor market regulations. Even so, we believe that our model is still predictive of the reality, because firms can resort to various margins that are essentially equivalent to employment adjustment: for instance, firms still retain the right to determine temporary employment; and they will be alternatively more selective at hiring, so job-finding rates are lower, probably causing a similar effect on the unemployment rate.[5]

This paper is organized as follows. The next section describes our general equilibrium model with UI and presents its key features. Section III presents the optimal UI problem and derives some basic properties of optimal design through conducting a calibration analysis. The final section summarizes the main results of the paper.

II. The Model

1. Environment

The workforce of N identical workers with time endowment T faces one of the following two states: employment (state 1) or unemployment (state 2). For state 1, labor supply is

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high productivity/high risk jobs. However, there is no study that focuses on the effect of UI on "work effort."

[4] See Johnson and Layard (1986) for a survey of modern unemployment and the policies for its reduction. Nickell and Layard (1999) offer a more recent survey of the causes of unemployment, which lends support to this paper's view on unemployment. Meyer (1990) provides empirical evidence of moral hazard in job search. This paper shares the general intuition in the literature that generous unemployment benefits exacerbate unemployment and reduce incentives to work. In U.S., before the 1996 welfare reform, AFDC recipients may fall into our definition of unemployment due to a generous provision of broadly defined benefits.

[5] In most OECD countries with regulations on layoffs, youth unemployment is also severe. Also, in many countries including Spain and Korea, temporary jobs constitute a large portion of employment in response to employment protection regulations.

"indivisible," as in Hansen (1985) and many other macroeconomics papers, such that workers should stay in their firms for a required amount of time L in accordance with the employment contract (e.g., nine to five o'clock each day). But they can choose their own effort level e while taking the risk of being fired when they are caught shirking, i.e., $e < \underline{e}$, where $\underline{e}$ is the required threshold level of effort. In this case, the true working time becomes eL (effective labor supply), and $T - eL$ amounts to the usual concept of leisure (henceforth L will be normalized at unity). When caught shirking, workers are fired and receive unemployment benefits b. The risk of being fired is expressed by a continuous and twice-differentiable probability function $\pi(e,d)$, where d is the intensity with which a representative firm monitors workers incurring the per unit cost c; $\pi_e < 0$ and $\pi_d > 0$ for the usual case of $e < \underline{e}$, and further $\pi_{ee} \geq 0$, $\pi_{dd} \leq 0$ and $\pi_{ed} \leq 0$ apply; and $\pi_e = 0$ and $\pi_d = 0$ for the no moral hazard case of $e \geq \underline{e}$.[6] Accordingly, individuals are *ex ante* identical but *ex post* heterogeneous (i.e., either employed or unemployed), and the *ex post* worker types are revealed by their employment status. We consider a static model so that ex ante unemployment issue can be dealt with focus.

In our model, the social welfare maximizing government can choose the level of benefit b and also sets the level of t_w so as to finance the expenditures on public goods and UI (i.e., a balanced-budget constraint). When these features are combined with the imperfect information about individuals' effort level, some moral hazard is inevitable, which is a source of involuntary unemployment. Specifically, individuals choose their optimal effort and consumption C, knowing the structure of UI and the redistributive extent of the income tax. Other things being constant, as the redistributive nature of the income tax system and UI becomes stronger, the greater will be

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[6] The general form of $\pi(e,d)$ allows the risk of unemployment to also be affected by extraneous factors, e.g., a random job-destruction shock. In such a case, $\pi(e,d)$ has a certain constant term so that a random job loss can occur independent of moral hazard, e.g., $\pi(e,d) = \alpha_0 + h(e,d)$ where α_0 is a constant. For $e \geq \underline{e}$, π depends on the random job-destruction shocks only because moral hazard does not arise.

the *negative* work incentives under a general set of assumptions to be discussed later.

2. Consumer's Decisions

A representative individual's utility function is given by $U(C, T-e) + \psi(G)$ with standard assumptions, where public good G is separable from private goods, commodity C and leisure $T-e$.

The probability that a worker belongs to the state of employment (state 1) is $1 - \pi(e,d)$ and the counterpart for the state of unemployment (state 2) is $\pi(e,d)$. In state 1, the worker's problem is to make consumption choices with net earned income $(1-t_w)w$, while in state 2, the worker makes consumption choices with unemployment compensation b. Given the usual case of $w > b$, the income tax schedule here takes a simplistic form of progressivity if $t_w > 0$ (or regressivity if $t_w < 0$). Technically, we can view the utility maximization problem in the two-stage budgeting context. At the first stage, the worker determines the level of effort. The employment status is determined by effort, firms' monitoring, and possible random job destruction. Conditional on the employment status, the second stage determines consumption C. Given the sequential nature of our problem, we propose the following second-stage problem.

$$\text{State 1: } \max_{C^{em}} U(C^{em}, T-e) + \psi(G) \quad \text{s.t.} \quad C^{em} \leq (1-t_w)w, \tag{1}$$

where superscript *em* stands for employment (e.g., C^{em} is the consumption for the state of employment), t_w is the income tax rate, and w is the wage (the total compensation paid by the firm). Given the standard properties of $U(\bullet)$, from this problem we can simply solve for the state-contingent demands for commodity C, which is given by $C^{em} = (1-t_w)w$. Similarly, the state 2 problem is defined for the unemployment state with superscript *un*:

State 2: $\max_{C^{un}} U\left(C^{un},T\right)+\psi(G)$ s.t. $C^{un} \le b$.   (2)

From this, $C^{un} = b$ is derived.

Once the state-contingent Marshallian commodity demands are obtained from the second stage, effort supply is determined by solving the first-stage problem. To simplify the problem, we substitute the commodity demands C^{em} and C^{un} into the direct utility U. Then, the utility for an employed worker, $U\left(C^{em},T-e\right)$, is denoted as $V^{em}\left((1-t_w)w,T-e\right)$, a quasi-indirect utility "conditional on leisure"; and the counterpart for an unemployed worker is $V^{un}\left(b,T\right)$. Then the first-stage problem is given by:

$$\max_{e} \left\{(1-\pi(e,d))V^{em}\left((1-t_w)w,T-e\right)+\pi(e,d)V^{un}\left(b,T\right)\right\}+\psi(G).$$ (3)

In this problem, the effort level is chosen after considering not only the utility from leisure but also the chance of unemployment and fiscal policy parameters. The first-order condition (henceforth, FOC) with respect to e is:[7]

$$\pi_e(V^{em}-V^{un})+(1-\pi(e,d))V^{em}_{T\text{-}e} = 0.$$ (4)

From this, we can define $e = e((1-t_w)w,b,d)$. Usual efficiency wage models assume some properties for the effort function, such as $\partial e/\partial w > 0$, $\partial e/\partial t_w < 0$, $\partial e/\partial d > 0$, etc.[8] We will see later that in the usual set of assumptions, these results hold.

Comparison with Baily (1978)

       Our formulation of consumers' problem contrasts with that of Baily (1978) where the

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[7] The utility function is assumed to be concave with respect to effort: $-\pi_{ee}(V^{em}-V^{un})+2\pi_e V^{em}_{T\text{-}e}+(1-\pi)V^{em}_{T\text{-}e,T\text{-}e} < 0$, so that the second-order condition holds.

utility function takes the form of $p(e)U^{em}(C^{em}, T-e) + (1-p(e))U^{un}(C^{un}, T-e)$. First, although it appears not much different from our formulation given in (3), the focus is totally different: it shows that an unemployed worker can increase the "job-finding" chance p by searching harder with a higher e, independent of states. Second, firms' endogenous responses including detection intensity d are not considered while possible in our model to be discussed later. Third, as a result, general equilibrium interactions between workers and firms cannot be studied while they can be in our model to be discussed later. Fourth, in a context related to the former argument, the efficiency cost of running a UI program cannot be studied while it is possible in our model to be discussed later. Other studies that adopt search-matching models also take an ex post approach similar to Baily (e.g., Davidson and Woodbury, 1997). Hopenhayn and Nicolini (1997) take a contract approach but again focuses on unemployed workers so that can be said to take an ex post approach. Overall, the existing UI literature takes UI from the perspective of unemployed workers. Therefore our ex ante approach can shed light on the optimal UI in a different angle at the very least.

3. *Producer's Decisions*

Following the standard assumption of the constant returns to scale (CRS) technology, we adopt the simplest possible form in that category: output is produced with a single production factor, effort, and the production function is $f(e) = e$. Here only the effort exerted by the employed workers $N(1-\pi(e,d)) \cdot e$ leads to production, since other workers are fired and live on benefits. A large number of identical firms act as Nash competitors: firm i with n_i workers chooses its wage and detection levels using the knowledge of the effort function of workers in order to maximize its profits, taking as given the wages and detection levels at other firms as follows.

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[8] These results hold for the usual utility functions such as CES and King-Rebelo-Plosser utility functions.

$$\max_{w,d} \left[(1-\pi(e,d))(e-w)-cd\right]n_i \quad \text{s.t.} \quad e = e\big((1-t_w)w,b,d\big), \tag{5}$$

where the firm pays wage w only to employed workers while incurring detection costs for all workers. Profit maximization leads to the following FOCs with respect to the choice variables w and d:[9]

$$-\pi_e \cdot e_w(e-w) + (1-\pi)(e_w-1) = 0 \text{, and} \tag{6-1}$$

$$-\pi_e \cdot e_d(e-w) - \pi_d(e-w) + (1-\pi)e_d - c = 0. \tag{6-2}$$

Using the shorthand, $g \equiv (1-\pi)(e(\bullet)-w)$, we re-express equations (6-1) and (6-2) as $g_w\big((1-t_w)w,b,d\big) = 0$ and $g_d\big((1-t_w)w,b,d\big) - c = 0$, respectively. From $g_d(\bullet) - c = 0$, the detection function $d = d\big((1-t_w)w,b,c\big)$ can be defined based on the implicit function theorem. Discussions on its properties are possible after introducing specific assumptions in subsection II.5 later.

Firms' output is eventually distributed to workers, firms and the government in the forms of wages, profits, and taxes, respectively, which comprises the demands for goods. Accordingly, equilibrium in the goods market is given by:

$$\begin{aligned} N\big[(1-\pi(e,d))\cdot e - \alpha\big] &= N(C+cd) + G \\ &= N\big[(1-\pi(e,d))C^{em} + \pi(e,d)C^{un} + cd\big] + G, \end{aligned} \tag{7}$$

where α is the profit of a representative firm; and the units of goods are normalized such that the rates of transformation among two goods, C and G, are unity. This can also be called the resource constraint, which relates aggregated output of individual firms to the total consumption by workers.

4. Government's Budget Constraint

The last equation for our model is about the government's balanced-budget constraint

(henceforth, GBC), denoted as $R(t_w, b; G, N) = 0$. The government can collect actuarially fair income tax to finance the expenditure on public good G and the unemployment benefits paid to jobless workers:

$$N(1 - \pi(e,d))t_w w = G + N\pi(e,d)b \tag{8}$$

5. Equilibrium

Given benefits and taxes, we can define the general equilibrium with five endogenous variables $\{C^{em}, C^{un}, w, e, d\}$ that can be characterized by the following equations describing (i) the two state-contingent commodity demands derived from the second-stage problem of the representative individual (see (1) and (2)), (ii) the FOC for individual effort (equation (4)), and (iii) the two FOCs for wage and detection (equations (6-1) and (6-2)). In what follows, we will examine the optimal taxation problem in which firms' profit α is zero, so that we can abstract from the issues such as bankruptcy of firms or uses of extra profits:

$$(1 - \pi)(e - w) - cd = 0. \tag{9}$$

Under this condition, we obtain the resource constraint RC of the economy, as follows:

$$N[(1 - \pi(e,d)) \cdot e] = N[(1 - \pi(e,d))C^{em} + \pi(e,d)C^{un} + cd] + G. \tag{7'}$$

Equilibrium unemployment and output

Using the incentive compatibility conditions, the FOC for the individual effort (4) combined with the two FOCs for firms' w and d (6-1) and (6-2), we can express the equilibrium levels of effort, wage, and detection as functions of the policy variables of income tax and benefit and other exogenous variables G and c, and then call them the "indirect" functions: $e^*(t_w, b; G, c)$,

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[9] The second-order conditions are assumed to be satisfied. In a later calibration, we will show that it is satisfied.

$w^*\left(t_w, b; G, c\right)$ and $d^*\left(t_w, b; G, c\right).$[10] Using them, we can study the responses of endogenous variables to taxes and benefits around the social optimum.[11]

Now, we can define "equilibrium unemployment" under a given set of taxes and benefits as the job separation rate, π:

$$u\left(t_w, b; G, c\right) = \pi\left(e^*\left(t_w, b; G, c\right), d^*\left(t_w, b; G, c\right)\right), \tag{10}$$

where $\pi_e < 0$, and $\pi_d > 0$. Proxying the unemployment rate by the job separation rate is inevitable in our static model. In fact, this is not so restrictive because even dynamic job-matching models possess the feature that a greater job-separation rate leads to a higher unemployment rate for a given job-matching technology (see Pissarides, 2000). Next, the output *Y* corresponding to the equilibrium unemployment is defined as:

$$Y\left(t_w, b; G, c\right) = eN(1 - \pi(e, d)) = eN\left(1 - u\left(t_w, b; G, c\right)\right). \tag{11}$$

Moral hazard arising from UI benefit and income tax

In our model an increase in benefit should be financed by an increase in the income tax rate. To see how the benefit affects the unemployment rate around the equilibrium, we first decompose the benefit effects on unemployment using the equilibrium unemployment equation (10). Appendix A shows that under some usual assumptions, we can obtain the following results summarized by Result 1.

Result 1. Under a set of "usual" assumptions given in Appendix A, the following set of partial

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[10] Treating *e, w,* and *d* as endogenous variables, we can express them as functions of policy variables, t_w, b and exogenous variables *G, c*. For instance, $e^*\left(t_w, b; G, c\right) = e\left((1 - t_w)w^*(t_w, b; G, c), b, d^*(t_w, b; G, c)\right)$.

[11] Note that in this case, both the government budget constraint and the resource constraint are satisfied around the social optimum, but they are not satisfied far away from the optimum.

derivatives holds: $e_{t_w}^* < 0$, $e_b^* < 0$; $d_{t_w}^* > 0$, $d_b^* > 0$.

Proof. See Appendix A. ▪

Using these derivatives in Result 1, we can replicate the empirical regularity that has been established through vast empirical studies: high UI benefits lead to an increase in unemployment. While the main result is identical to that of existing studies, our model shows that this moral hazard result is based on new elements of (i) employed worker's moral hazard, and (ii) the interaction between individuals' and firms' responses combined with (iii) the general equilibrium effect arising from the income tax financed UI system:

$$\frac{du}{db} = \pi_e \underbrace{\frac{de^*(t_w,b;G,c)}{db}}_{\text{(i) individual}} + \pi_d \underbrace{\frac{dd^*(t_w,b;G,c)}{db}}_{\text{(ii) firm}} \tag{12}$$

$$= \pi_e \left(\frac{\partial e^*}{\partial b} + \underbrace{\frac{\partial e^*}{\partial t_w}\frac{dt_w}{db}}_{\text{(iii-1) tax financing effect 1}} \right) + \pi_d \left(\frac{\partial d^*}{\partial b} + \underbrace{\frac{\partial d^*}{\partial t_w}\frac{dt_w}{db}}_{\text{(iii-2) tax financing effect 2}} \right)$$

$$= \underbrace{\pi_e}_{(-)} \left(\underbrace{e_b^*}_{(-)} + \underbrace{e_{t_w}^*}_{(-)} \underbrace{\frac{dt_w}{db}}_{(+)} \right) + \underbrace{\pi_d}_{(+)} \left(\underbrace{d_b^*}_{(+)} + \underbrace{d_{t_w}^*}_{(+)} \underbrace{\frac{dt_w}{db}}_{(+)} \right) > 0.$$

III. Optimal Unemployment Benefits: quantitative analysis

1. The optimal UI problem

Based on the discussion so far, we can set up the optimal UI problem as follows. It can be viewed as an optimal fiscal policy problem with both expenditure and financing (tax) systems:

$$L(b,t_w,w,e,d) = \left\{ (1-\pi(e,d))V^{em}((1-t_w)w,T-e) + \pi(e,d)V^{un}(b,T) \right\} + \psi(G) \tag{13}$$

$$+ \lambda_1 \{ N[(1 - \pi(e,d)) t_w w] - G - N\pi(e,d) b \} \tag{GBC}$$

$$+ \lambda_2 \{ N[(1 - \pi(e,d)) \cdot e] - N[(1 - \pi(e,d)) C^{em} + \pi(e,d) C^{un} + cd] - G \} \tag{RC}$$

$$+ \lambda_3 \left(\pi_e (V^{em} - V^{un}) + (1 - \pi(e,d)) V_{T-e}^{em} \right) \tag{IC : UMC}$$

$$+ \lambda_4 \left(-g_w ((1 - t_w)w, b, d) \right) \tag{IC : PMC1}$$

$$+ \lambda_5 \left(c - g_d ((1 - t_w)w, b, d) \right) \tag{IC : PMC2}$$

where $C^{em} = (1 - t_w)w$; $C^{un} = b$; the specific forms of $g_w(\cdot)$ and $g_d(\cdot)$ are given in equations (7-1) and (7-2); λ_i 's for i=1,2,..,5 are the Lagrange multipliers.

It simply states that the government maximizes the social welfare under the set of constraints: the government's balanced budget constraint (GBC), the economy's resource constraint (RC), individuals' incentive compatibility condition (utility maximization, IC: UMC), and firms' incentive compatibility conditions (profit maximization, IC: PMC1 and PMC2). Given the non-linearity of first-order conditions, in what follows we characterize the optimal benefits by conducting a numerical analysis.[12]

2. Calibration

This section presents a numerical model and characterizes the optimal combination of UI benefit and the income tax rate. We propose a numerical model as follows. First, the preferences are described by Kimball and Shapiro's (2008) form of the King-Plosser-Rebelo utility function that is popularly used in macro quantitative studies:

$$U(C,e;\gamma,\eta,M,G) = \frac{C^{1-\gamma}}{1-\gamma} \left[1 + M\left(1 - \frac{1}{\gamma}\right) \frac{e^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right]^{\gamma} + \psi(G), \text{ with } \gamma > 0, \ M > 0, \ \eta > 0 \tag{14}$$

where γ is the relative risk aversion, M is the work aversion parameter, and η is the labor (effort) supply elasticity in usual models. This utility function is non-separable between

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[12] Unless we assume highly restrictive functional forms for $U(\bullet)$ and $\pi(\bullet)$, we cannot derive analytically tractable forms for optimal b and t_w.

consumption and effort supply, so in a sense we can characterize the properties of our model in a more general setting.

Next, we parameterize the probability of being fired and hence living on benefits with no work $\pi(\bullet)$ as follows:

$$\pi(e,d;\alpha_0,\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2) = \alpha_0 - \alpha_1 e^{\beta_1} + \alpha_2 d^{\beta_2} - \alpha_3 e\, d \text{, with}$$

$$\alpha_0 \geq 0, \ \alpha_1 > 0, \ \alpha_2 > 0, \ \alpha_3 > 0, \ 0 < \beta_1 < 1, \ 0 < \beta_2 < 1 \qquad (15)$$

where α_0 is the random job destruction rate; α_i for i=1,2,3 is the scale parameter; β_1 and β_2 are the shares of effort and detection, respectively. In this form of $\pi(e,d)$, the following usual properties hold: $\pi_{ee} \geq 0$, $\pi_{dd} \leq 0$ and $\pi_{ed} \leq 0$. For simplicity, we normalize $\alpha_0 = 0$, and N=1.

The detection cost function $dc(\bullet)$ is given by:

$$dc(d;c,\delta) = c \cdot d^\delta \text{, with } c > 0 \text{, and } \delta \geq 1. \qquad (16)$$

This functional form allows convexly increasing detection costs and includes the simple form of c times d, used in the theory section.

Parameter values are set as follows. Following the empirical literature, we set $\gamma = 2$ and η=0.5 as the benchmark relative risk aversion and elasticity of effort supply, respectively. The range of estimates of labor supply elasticity is a bit wide. Especially, macroeconomists tend to view η in the range of [1, 4] while micro-economists are rather opposite. As a benchmark result, we borrow the estimates from Lee (2001; 2008). For these two parameters, we need some sensitivity checks, which will be conducted later. Next, other parameter values are chosen to help satisfy the following set of the data: (i) the sample rate of workers who are either unemployed or living on welfare is 18.7% which is close to the notion of π in our model;[13] and (ii) the size of

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[13] This estimate is based on the U.S. data as follows. For the U.S., the average unemployment rate of 5.59% in 1990-2000, combined with about 10 million AFDC recipients in the early 1990s, translates into the estimate that about

government *GS*, defined as $(G + \pi b)/Y$, is chosen to be about 41% of output, based on the OECD data base. Given that there is no empirical evidence on $\pi(\bullet)$, we set them at $\{ \alpha_1 = \alpha_2 = 0.434, \alpha_3 = 0.0015, \text{ and } \beta_1 = \beta_2 = 0.5 \}$ as a starting set of parameter values. To make the ratio of benefit recipients to the potential workforce close to the data, we set the value of *M* = 1.0. *G* is set at 0.128 to satisfy the observed share of public expenditures including social insurance. For the parameters of the detection cost function $dc(\bullet)$, we set $c = 0.011$, and $\delta = 2$ to express convexly increasing detection costs. We will check other sets of parameter values for sensitivity of our results.

3. Simulation Results

The base case numerical results

Under the base case set of parameter values, we obtain the welfare-maximizing optimal equilibrium where the values of endogenous variables and policy variables *b* and t_w are determined. The base case simulation results are given in the top panel of Table 2. Using the calibrated parameter values, we obtain the results on key variables: {*b*=0.159, $t_w = 0.428$, *e*=0.474, *w*=0.453, *d*=1.254, π =0.187, *GS*=0.409}. In the base case, we find that the replacement rate using the before-tax income, *b/w*, is 0.352. This figure may be somewhat lower than the typical results that are usually obtained from the studies taking the usual ex post approach; and the after-tax replacement rate is 0.616, just a bit higher than the OECD average replacement ratio defined using the before-tax incomes.[14] This low replacement rate is partly because our model takes an ex ante approach -- it focuses more on reducing the source of unemployment by boosting effort.

12.7% of the potential labor force is not working and is living on benefits. At the average unemployment rate of 8.7% for welfare states in the sample period, the corresponding figure is imputed to be about 18.7% for our sample.
[14] The replacement rate of the OECD sample is about 58.4%.

The income tax rate of 42.9% seems close to the level in welfare states, although a direct comparison with the data might be misleading because the current model setting does not consider heterogeneity in ability and redistribution from high- to low-ability individuals. Looked at another way, full insurance with a 100% income tax rate is not optimal due to moral hazard. The size of government GS, defined as $(G + \pi b)/Y$, is close to the OECD data base.

Furthermore, in the benchmark case, we can see that the assumptions used in derivation of Result 1 hold (see the lower panels in Table 2). (i) The usual assumptions from most efficiency wage models hold indeed, e.g., $e_w > 0$, $e_{t_w} < 0$, $e_b < 0$, $e_d > 0$, $e_{d,t_w} > 0$, $e_{d,w} < 0$; (ii) we can also verify the assumption that partial derivatives of d with respect to taxes and benefits have signs opposite to those of e; and (iii) the last assumption that the signs of the partial equilibrium effects of taxes and benefits on e and d are consistent with their general equilibrium counterparts is found to hold, as we see in the "total effects" row in the middle panels of Table 2. The results of benefits and tax effects in the base case equilibrium therefore support the conclusion in Result 1. These results combined together suggest that the theoretical results can be supported by our numerical model with more general features.

Comparative statics analyses

We present how the optimal levels of UI benefit and the income tax rate respond to changes in parameters of the model. First, the optimal UI benefit b^* falls as the labor supply elasticity η goes up. This is due to the moral hazard problem. Because a high labor supply elasticity implies a sensitive behavioral response to incentives, UI benefit should be low to keep moral hazard low in equilibrium.

Second, the optimal UI benefit should go up as the relative risk aversion γ goes up,

because individuals get more risk-averse and are therefore more eager to stabilize consumption. In this case, holding other things constant, they want to boost the purchasing power of unemployment benefits in preparation for possible unemployment. However, at the conventionally accepted range of estimates of γ, the UI benefits do not change very much. This is somewhat in contrast to the results from the existing ex post studies. Obviously, other things being constant, a high UI benefit combined with a high income tax rate can boost welfare by providing better insurance in the sense of Varian (1980). But in our ex ante approach the work incentives are particularly important, so the insurance effect should balance with the work incentive effect. For this reason, even for a high value of risk aversion, the optimal UI benefit is not very high.

Third, the size of public goods expenditure or the size of pre-existing distortions G is also a crucial element that affects the level of benefits. With a high benefit, the marginal efficiency cost of public fund (MCPF) is high, so that we can afford only a low benefit to keep the balance between incentive and insurance effects. This is what we cannot address in the usual ex post approach.

There are many other cases of interest, but they are not reported in the text for brevity. Overall, our numerical model confirms the predicted signs of various partial derivatives and characterizes the model's behaviors at the social optimum.

Identifying firms' endogenous responses to policy changes

As UI benefits rise, the equilibrium unemployment rate u rises in our model, and this theoretical result is numerically validated as we see in Table 4. To account for how much of this rise is due to individuals' and firms' responses, respectively, we do the following experiment. First, after fixing the firm's detection level at the benchmark equilibrium level, $d = d^*$ as if d is

exogenous, we calculate the hypothetical equilibrium with a 10% increase in benefit (see the row for exogenous d in Table 4). Second, in a similar way we calculate the equilibrium in response to the same 10% increase in benefit *without* any restriction on d (see the row for endogenous d). Third, we compare these two different equilibria.

As we see in Table 4, the result from this exercise shows that the unemployment rate 0.196 for the hypothetical case is much lower than 0.223, the one that we obtained with d determined endogenously. It suggests that firm's behavioral change is important, unlike the usual UI models where only the individual's behavioral responses are studied. In the lower panel, we can see that the moral hazard from benefit and income taxation goes down under the case of endogenous d, generating the pattern that the negative responses to benefit and income taxation attenuate due to a rising d.

Sensitivity analysis

To give a sense of equilibrium at the points that are substantially different from the base case values, we show in Table 5 the results from the sensitivity analysis on the key variables: benefit, the income tax rate, effort, wage, detection, the unemployment rate, and output.

Not surprisingly, the results vary substantially with changes in exogenous parameters, but the qualitative pattern of the changes obtained from comparative statics still holds. In particular, firms' endogenous employment adjustment through d accounts for an important variation in changes in the unemployment rate.

Through varying the risk aversion parameter γ from 0.5 to 3.0 (see the first row in Table 5), we find that γ is an important determinant of the optimal UI benefit level, but it produces little variation in the replacement ratio, b/w, which contrasts with the conclusion of the studies adopting the usual ex post approach.

Some discussion about the labor supply elasticity may be warranted (see the second row in Table 5). At a low value of $\eta = 0.2$, effort supply does not respond much to incentives and moral hazard is basically not an issue. In this case, firms do not need to strengthen monitoring. Therefore the government can improve welfare by providing a bit better insurance, which requires a bit higher benefit with a low income tax rate. This point can be supported by a high replacement ratio, b/w, well above 0.5. At a high value of $\eta = 1.0$ or 4.0, a value typically assumed in macro business cycle studies, compared to the benchmark, moral hazard can potentially be severe because effort supply gets more sensitive to incentives. The resulting optimal replacement ratio is very low, less than 0.2. At the new social optimum, therefore, the government provides a low benefit, and firms increase monitoring intensity and unemployment rises as a result. This is the result we cannot obtain in partial equilibrium models.

The size of public good expenditure G is an important component of the optimal UI benefit level. As the panel for G shows, the cost of a social insurance program rises with G, functioning as a pre-existing distortion. Therefore the standard marginal cost of public fund (MCPF) argument applies to the current problem. Since G affects the utility of both employed and unemployed workers symmetrically, there is no distortion in the expenditure side. However, to finance a greater expenditure for G, we should tax labor income heavily, which causes distortions in effort supply. The optimal policies for this case should consider efficiency with a larger weight than insurance. The resulting policy combination is therefore to lower benefit and raise the income tax rate simultaneously such that no behavioral changes occur in real activities including e, w, d, u and Y (see the rows for G in Table 5). The replacement ratio also falls. This may be a textbook result but the usual ex post approach cannot address this issue due to lack of general equilibrium features.

Next, a greater random job destruction rate α_0 requires a larger UI expenditure, which

needs to be financed by a higher income tax rate. To fight a possible decline in work effort, the optimal benefit level b falls slightly despite the need for a greater insurance. Again, this is largely due to general equilibrium effects. When δ, the power to detection, rises to 2.5, the detection becomes costly, and thus firing and unemployment fall as a result, along with low effort and wage. Because the UI expenditure gets lower because of low unemployment, the optimal benefit level can go up in equilibrium.

For brevity, we skip detailed discussion about changes in other parameter values, but we verify that the basic pattern from the comparative statics holds across other sets of parameter configurations.

VI. Summary and Conclusion

In discussion of the optimal unemployment insurance system, the literature (e.g., Baily, 1978) tends to focus on unemployed workers' moral hazard in job search conditional on "unemployment that has already occurred." In contrast, this study considers the ex ante optimal UI "before unemployment occurs" from the perspective of workers on job. We look not only at moral hazard in work efforts of workers on job caused by generous UI benefits but the resulting firms' employment adjustment in response to worker moral hazard. Advantages of this ex ante approach include that (i) we can study the interaction between individuals and firms in response to UI benefit changes, and (ii) we can design the optimal UI in the context that prevents the occurrence of unemployment, advancing the traditional notion of optimal UI.

We propose a simple general equilibrium model where workers choose effort supply and firms set wages and adjust employment. Unlike existing studies, our analysis shows that firms reduce employment in response to changes in workers' behavior arising from UI benefit increases. This endogenous response of firms potentially accounts for more than a half of the total changes

in unemployment in response to UI benefit changes. By conducting a numerical analysis, we find that (i) labor supply elasticity is a central determinant of the optimal benefit level, more important than the risk aversion parameter; (ii) the optimal benefit level is also sensitive to the size of existing public expenditures; and (iii) the optimal level is lower than those in existing studies.

These new findings are, of course, based on the new, ex ante approach taken in this paper. Nevertheless, we believe that the main conclusions obtained from both ex post and ex ante approaches should be taken into account in the design of the optimal UI structure in reality. For instance, the optimal benefit level may be bounded between the typical estimates based on ex ante and ex post approaches, e.g., [0.35, 0.5].

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Table 1. The Parameters

parameter	definition	calibrated value
γ	The relative risk aversion	2.000
η	The effort supply elasticity	0.500
M	The work aversion parameter	1.000
α_0	The baseline probability of job destruction	0.000
α_1	The scale parameter for e in the probability function π	0.434
α_2	The scale parameter for d in the probability function π	0.434
α_3	The scale parameter for interaction term for e and d in π	0.0015
β_1	The share of e in the probability function π	0.500
β_2	The share of d in the probability function π	0.500
c	The coefficient in the monitoring cost function	0.011
δ	The power to the monitoring cost function	2.000

Table 2. The Base Case Results

	b	t_w	e	w	C^{em}	C^{un}	d	π/u	U^{em}	U^{un}	$GS^\dagger$
Simulation	0.159	0.428	0.474	0.453	0.259	0.159	1.254	0.187	-4.003	-6.275	0.409
Data	---	---	---	---	---	---	---	0.187	---	---	0.409

▪Checking signs of derivatives
Partial effects*

	e_b	e_w	$e_{w,w}$		e_{t_w}	e_d	$e_{d,d}$	e_{d,t_w}	$e_{d,w}$
e	-2.820	0.992	-3.220		-0.786	0.039	-0.009	0.029	-0.036

	d_b	d_{t_w}	d_e
d	6.411	1.788	-6.736

	$\pi_{e,e}$	$\pi_{d,d}$	$\pi_{e,d}$
π	0.332	-0.077	-0.002

▪Total effects**

	e_b^*	$e_{t_w}^*$
e	-1.255	-0.364

	d_b^*	$d_{t_w}^*$
d	10.136	2.933

▪Second-order conditions (SOC)
individual's utility maximization for e: -4.431, satisfied
firm's profit maximization for w and d: 0.029, satisfied

Notes. *: Partial effects refers to the partial derivatives that are evaluated at the base case equilibrium. **: Total effects refers to the overall effects, taking into account both the partial derivatives and general equilibrium feedback effects. $\dagger$: $GS = (G + \pi b)/Y$, a usual measure of government size.

Table 3. Comparative Statics at the Base Case Equilibrium

Variable	base case	↑γ	↑η	↑G	↑M	↑α_1	↑α_2	↑α_3	↑β_1	↑β_2	↑c	↑δ
b	0.159	0.167	0.148	0.151	0.150	0.172	0.150	0.159	0.164	0.155	0.161	0.163
b/w	0.352	0.353	0.331	0.333	0.344	0.372	0.324	0.352	0.350	0.341	0.358	0.361
$b/(1-t_w)w$	0.616	0.602	0.588	0.616	0.616	0.619	0.615	0.616	0.612	0.614	0.616	0.618
t_w	0.428	0.414	0.437	0.459	0.441	0.398	0.474	0.429	0.428	0.444	0.418	0.416
e	0.474	0.495	0.472	0.474	0.457	0.482	0.489	0.475	0.491	0.480	0.471	0.469
w	0.453	0.472	0.448	0.453	0.437	0.461	0.462	0.453	0.468	0.455	0.450	0.451
d	1.254	1.285	1.304	1.254	1.232	1.268	1.332	1.261	1.288	1.336	1.178	1.161
π	0.187	0.187	0.197	0.187	0.188	0.156	0.247	0.188	0.198	0.207	0.172	0.170
C^{em}	0.259	0.277	0.252	0.245	0.244	0.277	0.243	0.259	0.267	0.253	0.262	0.263
C^{un}	0.159	0.167	0.148	0.151	0.150	0.172	0.150	0.159	0.164	0.155	0.161	0.163
U^{em}	-4.003	-4.085	-4.146	-4.229	-4.240	-3.740	-4.269	-4.007	-3.888	-4.096	-3.950	-3.934
U^{un}	-6.275	-7.162	-6.743	-6.628	-6.653	-5.825	-6.680	-6.281	-6.113	-6.434	-6.193	-6.153
Y	0.386	0.395	0.379	0.386	0.372	0.407	0.368	0.386	0.393	0.381	0.390	0.390
e_{t_w}	-0.785	-0.798	-0.788	-0.829	-0.774	-0.760	-0.869	0.787	-0.810	-0.811	-0.768	-0.766
d_{t_w}	1.778	1.767	1.856	1.878	1.795	1.716	2.016	1.769	1.628	2.094	1.656	1.620
SOC	yes*	Yes	yes	yes	Yes	yes	yes	yes	yes	yes	yes	yes

Notes: The increases in the listed parameters are 10% from the base case equilibrium. *: Second-order condition is checked and satisfied.

Table 4. Effects of endogenizing the firms' firing decisions d

▪Effects of a 10% increase in benefit

	π / μ	Y	Δd	$\Delta \pi$
Exogenous d	0.196	0.358	0.000	0.009
Endogenous d	0.223	0.353	0.158	0.036

▪Exogenous d vs. endogenous d

	e_b^*	$e_{t_w}^*$
Exogenous d	-1.835	-0.536
Endogenous d	-1.255	-0.364

Table 5.   Sensitivity Analysis

		b	t_w	e	w	d	π/u	Y
	base case	0.159	0.428	0.474	0.453	1.254	0.187	0.386
γ	0.5	0.117	0.524	0.368	0.352	1.080	0.187	0.299
	1.5	0.143	0.463	0.432	0.412	1.194	0.188	0.351
	3.0	0.202	0.346	0.596	0.570	1.380	0.174	0.493
η	0.2	0.329	0.298	0.580	0.570	0.916	0.084	0.532
	1.0	0.107	0.435	0.513	0.472	1.656	0.246	0.386
	4.0	0.101	0.259	0.989	0.863	2.822	0.293	0.699
G	0.0	0.244	0.124	0.474	0.453	1.254	0.187	0.386
	0.2	0.112	0.599	0.474	0.453	1.254	0.187	0.386
α_0	0.05	0.153	0.464	0.484	0.461	1.255	0.233	0.371
	0.10	0.145	0.450	0.494	0.469	1.256	0.280	0.355
δ	1.5	0.143	0.492	0.501	0.465	1.775	0.270	0.366
	2.5	0.166	0.404	0.464	0.448	1.071	0.153	0.393

Appendix A. Assumptions used in Result 1

In our general equilibrium model, we present the optimal UI structure through numerical analysis due to nonlinearity in key equations. Before conducting a numerical exercise, we show some general features of the model using some specific assumptions. To characterize the mechanism through which benefits and income taxation affect employment and other economic behaviors as intuitively as possible, in this appendix we introduce a set of simplifying assumptions. Although other cases are possible, we would like to confine our interest to the case (henceforth, the "usual" case) where all three of the assumptions are satisfied:

(i) functional assumptions: $\pi(e,d)$ has intuitive properties such as $\pi_{ee} \geq 0$, $\pi_{dd} \leq 0$ and $\pi_{ed} \leq 0$; [15] the utility function is assumed to be separable between consumption and leisure: $U_{C,T-e} = 0$;

(ii) partial derivatives of d with respect to taxes and benefits have opposite signs to those of e: $d_{t_w} > 0$, $d_b > 0$;

(iii) the signs of the partial equilibrium effects of taxes and benefits on e and d are consistent with their general equilibrium counterparts.

Using assumptions (i) and (ii), we can validate the following set of results about the effort function $e(.)$ (see Appendix B for proof): (a) the usual assumptions from most efficiency wage models hold ($e_w > 0$, $e_{ww} < 0$, $e_{t_w} < 0$, $e_b < 0$, $e_d > 0$, $e_{dd} < 0$, etc.); [16] and (b) $e_{d,t_w} > 0$, $e_{d,w} < 0$. Among the results in (b), $e_{d,t_w} > 0$ hold because given that tighter monitoring increases effort ($e_d > 0$), the monitoring effect diminishes when income tax rates

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[15] The properties of $\pi(e,d)$ are intuitive. $\pi_{ed} < 0$ is likely because the effect of a unit increase in detection falls as effort rises.

[16] These results can also be found in the literature (e.g., Pisauro, 1991; Agell and Lundborg, 1992; Shapiro and Stiglitz, 1984; Yellen, 1984).

decrease. This is because it is difficult to increase effort when it is already too high. $e_{d,w} < 0$ is consistent with the former results, implying that wages and monitoring are "substitutes" in eliciting effort. The roles of other assumptions will be mentioned below where necessary.

Assumption (ii) is based on the original motivation of this paper: firms strengthen monitoring to increase effort when the effort level is too low. If an exogenous factor leads to a decrease in effort, then profit-maximizing firms are usually better off with increasing monitoring, which accords with the motivation of this paper.[17]

Regarding assumption (iii), we believe that it is more interesting to discuss the effects of taxes and benefits in the usual cases where general equilibrium feedback effects would modify the size of the initial direct effects but do not dominate them.[18]

Appendix B: Properties of the Effort Function $e(.)$

Usual efficiency wage models show: (i) an increase in wage boosts effort, $\partial e / \partial w > 0$; similarly, an increase in the income tax rate will lead to a decrease in effort, $\partial e / \partial t_w < 0$; (ii) a more generous UI system leads to low effort, $\partial e / \partial b < 0$; (iii) a greater detection rate elicits more effort, $\partial e / \partial d > 0$. We need to check whether those assumptions are still valid in our model, and we also need to define the effect of commodity taxation on effort.

By applying the implicit function theorem to the FOC for e (equation (4)), we can derive the following properties of the effort function with respect to a change in variable j:

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[17] We could not determine the exact signs of the derivatives in assumption (ii) from applying the implicit function theorem to the FOC (6-2). However, the assumed case seems realistic, so we focus on the case where these conditions are met.

[18] For instance, our model assumes that generous benefits lead to low effort for a worker, $e_b < 0$, and that this is true in equilibrium also, i.e., $e_b^* < 0$. Of course, there is no a priori reason for this condition to hold, but such a case seems to be usual.

$$e_j \equiv \frac{\partial e}{\partial j} = -\frac{\pi_e(V_j^{em} - V_j^{un}) + (1-\pi)V_{T-e,j}^{em} + \pi_{e,j}(V^{em} - V^{un}) - \pi_j V_{T-e}^{em}}{A}, \qquad \text{(B-1)}$$

where $j = w, b, d, t_w$; and $A \equiv \pi_{e,e}(V^{em} - V^{un}) - 2\pi_e V_{T-e}^{em} - (1-\pi)V_{T-e,T-e}^{em} > 0$. Making use of the

assumptions: $U_{C,T-e}^{em} = 0$ (i.e., $V_{T-e,j}^{em} = 0$), $\pi_{ee} > 0$, $\pi_{dd} < 0$ and $\pi_{ed} < 0$, we can obtain:

$$e_w \equiv \frac{\partial e}{\partial w} = -\frac{\pi_e V_w^{em}}{A} > 0, \qquad \text{(B-2-1)}$$

$$e_b \equiv \frac{\partial e}{\partial b} = \frac{\pi_e V_b^{un}}{A} < 0, \qquad \text{(B-2-2)}$$

$$e_d \equiv \frac{\partial e}{\partial d} = \frac{\pi_d V_{T-e}^{em} - \pi_{e,d}(V^{em} - V^{un})}{A} > 0, \qquad \text{(B-2-3)}$$

$$e_{t_w} \equiv \frac{\partial e}{\partial t_w} = -\frac{\pi_e V_{t_w}^{em}}{A} < 0, \qquad \text{(B-2-4)}$$

$$e_{d,t_w} = \frac{\left(\pi_{e,d} A - \pi_d V_{T-e}^{em} \pi_{ee}\right) V_{t_w}^{em}}{A^2} > 0, \qquad \text{(B-2-7)}$$

$$e_{d,w} = \frac{\left(\pi_{e,d} A - \pi_d V_{T-e}^{em} \pi_{ee}\right) V_w^{em}}{A^2} < 0. \qquad \text{(B-2-8)}$$

All the derivatives are determined with their expected signs. Cross-partial derivatives are derived

from differentiating equation (B-2-3) with respect to each variable of interest.